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Optimal Analysis of Flexible Reconfigurable Networks Using Movable and Changeable Components

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Abstract— **Most types of networks depend on fixed resources in their configuration, but these fixed resources are not effective, especially in the case of a problem in any of these resources. This may lead to a defect in the entire network until the problem is resolved and therefore there must be an alternative to replace the fixed resources in such cases. In this paper, the concept of addressing optimal operations of flexible reconfigurable networks with movable and changeable resources is thoroughly presented. The methodology is represented in a mathematical form by solving the optimization problems of quadratic programming, nonlinear programming and multi-objective programming. Implementation of the proposed technique is carried out to several illustrative examples with a single or multiple movable symbolic-based resources. The implemented examples demonstrate the efficacy of the proposed technique and its strong capability in presenting many scenarios for system operation under movable resources. Finally, the applications of the new concept could be extended to real life systems of the electricity, water, oil and gas, communication, computer, transportation, and service networks.**

Keywords— ***Flexible reconfigurable networks, Movable and changeable components, Parameter varying systems, Symbolic-based modeling and optimization.***

I. Introduction

The operation of real life physical systems undergoes continuous changes specially in their resources. Such resources are not all fixed in location or amount but they are better to have some of their components that are movable and changeable. The movable resource are defined as non-fixed type of resources which are not associated with a specific location , but they are flexible enough to move as a one entity or some of its position from one part of the system easily based on the requirement of the whole system as shown in **Fig. 1.** In general, the overall operations of such systems are very challenging as they require new types of mathematical formulation, analysis and solution.

The concept of "***reconfigurable system***" (which literally means the feature that the elements or settings of such system could be rearranged) is commonly introduced in computer networks to indicate their capability of reconfiguration upon requirement. One of the famous systems with such reconfigurability is the Field Programmable Gate Arrays (FPGAs), where the computer architecture is designed combining the flexibility of software operation with the adaptive high performance hardware [1]. The same concept has also extended by introducing the reconfigurable manufacturing systems that can produce different flexible process layouts depending on the changing requirements say of product volumes and mix types.

Other application of reconfigurable systems is in the area of flexible building design or responsive component architecture [2]. This flexible architecture permit the building components to be reconfigured based on changeable aspects of its affecting environment. In general, the concept provides powerful and flexible tools in system operation running in changing environment.

The notion of ***Flexible Reconfigurable Networks*** is a new approach that can be applied to sensor networks, water pumping networks, and power networks. In this case, we are optimizing the system and obtaining a general policy for operation. In such situation, we will be dealing with a typical parameter varying system represented by symbolic-based appropriate approaches [3-7].

There is no technique now available in the literature that can be tailored to handle this problem of the parameters varying nature. There are in fact limited work in the area of modeling real life systems with movable or changeable resources. There works were represented in the areas of telecommunication [8], sensor and actuator network[9, 10].

The main objective of this paper is to develop a mathematical-based methodology for policy development, rather than seeking a specific numerical solution of limited usage. The policy has to be derived upon maximizing certain overall gain generated from network, minimizing cost or provided shortages, ..etc.

II. Representation of Flexible Reconfigurable Networks

The representation of the flexible reconfigurable networks could be simply expressed by the small case study of 4-node network as shown in **Fig. 1**. In this figure, we have three fixed locations namely X, Y, and Z, with corresponding resources of x, y, and z respectively. In addition, we have one movable location denoted by M, with corresponding movable resources of m.

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In general the term "***Reconfigurable***" means to rearrange the elements or settings (after it was configured already) (this is the proposed approach that gives flexible systems that can be configured at any time based on the situations).

The various scenarios of this flexible network are shown in **Table 1**. The table provides the four different scenarios depicted in the **Fig. 2.**

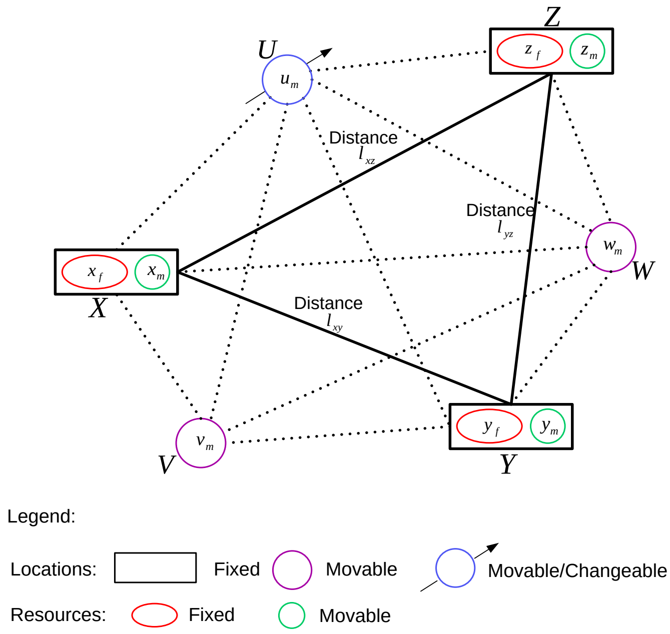


Fig. 1 A geographically distributed Flexible Reconfigurable Network with fixed and movable/changeable resources.

The same representation can also be extended to networks having multiple movable and changeable resources m_1, m_2 ,..,etc. These movable components are treated in the mathematical formulation in a separate way to enable the flexibility of their movement within the overall network.

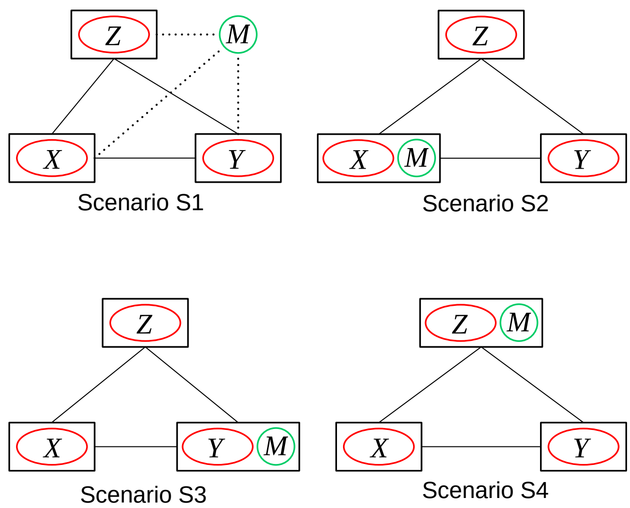


Fig. 2 Various scenario of 4-node Flexible Reconfigurable Network by manipulating a single movable component.

TABLE 1 Various scenarios of the 4-node Flexible Reconfiurable Network described in **Fig. 2**.

Scenario name	Available overall potential resources			
	Fixed locations			Movable location
	X	Y	Z	M
S1	x	y	z	m
S2	x + m	y	z	0
S3	x	y + m	z	0
S4	x	y	z + m	0

III. CLASSIFICATIONS OF FLEXIBLE RECONFIGURABLE NETWORKS

The various classifications of flexible reconfigurable networks with movable/changeable components are very wide and can include the majority of real life operational network specially of the geographical distributed types. Nevertheless, the classification will be limited only to the combination of the network nodes location as fixed location and movable locations. Then for each location, we have three different components of resources fixed or changeable. This gives the various alternatives described in TABLE 2.

In general, the complexity of these problems will be related in the first place on the number of the parameter varying component in the system where the symbolic representation and solution approaches are applied. Other fixed components are in fact of the numeric type that can be handled by the conventional computational techniques. Moreover, for the simplicity of the work only Classes II to VI are considered which combinations of fixed and movable resources are. Classes VII and VIII will be postponed and could be regarded only within the future research work.

TABLE 2 Various classifications of Flexible Reconfigurable Networks.

Class number	Fixed location resources types		Movable location resources Types	
	Fixed resources	Changeable resources	Fixed resources	Changeable resources
I	√			
II	√	√		
III	√		√	
IV	√	√	√	
V	√		√	√
VI*	√	√	√	√
VII			√	√
VIII				√

***** A typical example of this class is shown in **Fig. 2.**

During the implementation, the movable resources are kept in symbolic form in the investigation. Such symbolic-based representation usually lead to a generic exact solution to the problem where a general policy could be attained and applied in a flexible manner by manipulating the available flexible movable/changeable components. In most situations, the formulation of these networks towards seeking optimal operation solution will be based on matrix representation and manipulation. Such matrix-based formulation could be handled by appropriate symbolic computation software tools.

In the following sections, the notion of optimal operation of flexible reconfigurable networks is implemented through solving three applications handled by quadratic, nonlinear and multi-objective optimization.

IV. IMPLEMENTATION USING QUADRATIC PRGRAMMING

A. *Mathematical Formulation*

The general form of Quadratic Programming optimization problem with movable resources m_i can be represented as follows [11, 12]:

$$\begin{aligned} &\text{Minimize} \quad cx + \tfrac{1}{2}x^T Q x \\ &\text{Subject to} \quad Ax = \\ &b\begin{cases} A_i x + m_i \le b_i & \text{where } i = 1,2,\dots,r \\ A_j x - m_j \ge b_j & \text{where } j = 1,2,\dots,n \end{cases} \end{aligned} \tag{1}$$

Let us define:

$$x = (x_1 \quad x_2 \quad \cdots \quad x_n)^T \tag{2}$$

$$c = (c_1 \quad c_2 \quad \cdots \quad c_n) \tag{3}$$

$$Q = \begin{pmatrix} q_{11} & q_{12} & \cdots & q_{1n} \\ q_{21} & q_{22} & \cdots & q_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ q_{n1} & q_{n2} & \cdots & q_{nn} \end{pmatrix} \tag{4}$$

where Q is an (n x n) symmetric matrix. Also, we let:

$$A = \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{pmatrix} \tag{5}$$

and

$$b = \begin{pmatrix} b_1 - m_i \\ b_2 + m_j \\ \vdots \\ b_m - m_i \\ b_{m+1} + m_j \end{pmatrix}. \tag{6}$$

Applying Lagrange's theorem to the above formulation, the optimal solution x^* can be expressed as [9]:

$$x^* = Q^{-1}c^T + Q^{-1}A^T(AQ^{-1}A^T)^{-1}(b - AQ^{-1}c^T). \tag{7}$$

Equation (7) can be solved using Symbolic-based Matlab Software "MuPad" keeping the movable parameters as symbols at all steps of solution [13].

It is pointed out that for the constrained quadratic programming problems and its similar approaches the values at the RHS of their formulations are related directly or indirectly to the problem resources, which is the terms to be manipulated by the movable/changeable components of the suggested approach.

B. *Illustrative Example*

We will demonstrate now the concept of introducing the movable resources facilities with the original fixed unchangeable facilities by solving the following typical quadratic programming **Illustrative Example #1**.

Original Problem #1:

The original problem without movable resources can be expressed as:

$$\begin{aligned} &\text{Minimize} \quad f(x) = x_1^2 + x_2^2 \\ &\text{subject to} \quad g_1(x) = 6x_1 + x_2 = 6 \\ &\qquad g_2(x) = x_1 + 5x_2 = 1 \\ &\qquad g_3(x) = 2x_1 + 4x_2 = 8 \\ &\qquad g_4(x) = -3x_1 + -3x_2 = 1. \end{aligned} \tag{8}$$

The original feasible operation zone of (8) obtained by plotting x_2 *versus* x_1 togerther with the objective function are shown in **Fig. 3**.

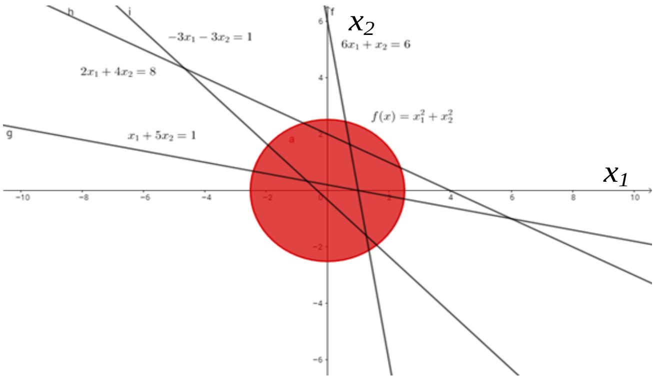


Fig. 3 Plot of x_2 *versus* x_1 for the original **Illustrative Example #1** without movable components.

Modified Problem #1:

The modified problem #1 after adding two movable resource m_1 *and* m_2 , can be witten as:

$$\begin{aligned} &\text{Minimize} \quad f(x) = x_1^2 + x_2^2 \\ &\text{subject to} \quad g_1(x) = 6x_1 + x_2 = 6 - m_1 \\ &\qquad g_2(x) = x_1 + 5x_2 = 1 + m_1 \\ &\qquad g_3(x) = 2x_1 + 4x_2 = 8 - m_2 \\ &\qquad g_4(x) = -3x_1 + -3x_2 = 1 + m_2. \end{aligned} \tag{9}$$

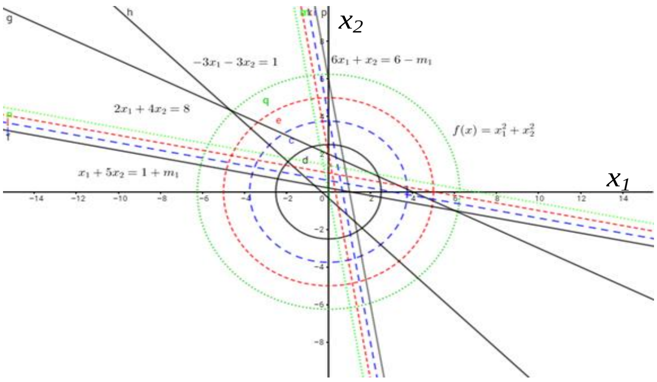
The modified feasible operation zone of (9) obtained by plotting x_2 *versus* x_1 togerther with the objective function are shown in **Fig. 4(a)** for changeable m_1 and fixed m_2 and **Fig. 4(b)** for fixed m_1 and changeable m_2 .

The optimal solution $x^*$of the problem can be expressed as follows:

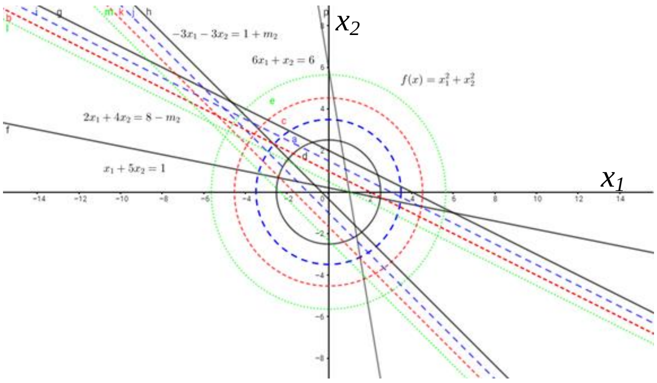
$$x^* = Q^{-1}c^T + Q^{-1}A^T(AQ^{-1}A^T)^{-1}(b - AQ^{-1}c^T) \tag{10}$$

Or after substitution with the terms of $c^T = 0$,we obtain:

$$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}^{-1} \begin{pmatrix} 6 & 1 \\ 1 & 5 \\ 2 & 4 \\ -3 & -3 \end{pmatrix}^T . \left[\begin{pmatrix} 6 & 1 \\ 1 & 5 \\ 2 & 4 \\ -3 & -3 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}^{-1} . \begin{pmatrix} 6 & 1 \\ 1 & 5 \\ 2 & 4 \\ -3 & -3 \end{pmatrix}^T \right]^{-1} . \begin{pmatrix} 6 - m_1 \\ 1 + m_1 \\ 8 - m_2 \\ 1 + m_2 \end{pmatrix}. \tag{11}$$



a) Changing of m_1 and fixed m_2 resources.



b) Fixed m_1 and changing of m_2 resources.

Fig. 4 Plots of x_2 *versus* x_1 for the modified **Illustrative Example #1** with movable components for two changing scenarios.

Equation (11) can be solved and the optimal solution of x_1 *and* x_2 can be determined as function of the symbolic movable resources m_1 and m_2 .Examples of the optimization results by separately varying m_1 and m_2 resources is shown in TABLE 3 (a,b).

The overall effect of changing both m_1 *and* m_2 on the objective function $f(x)$ is plotted in **Fig. 5**. In the investigation both m_1 *and* m_2 are assumed to vary in increment of "1" in the range from 0 to 6 and 8 units respectively.

TABLE 3 Effect of separate changes of m_1 and m_2 resources on the objective function $f(x)$ results.

a) Changing of m_1 and fixed m_2 resources.

m_1	0	1	2	3	4	5	6
x_1	2.25	2.75	3.25	3.75	4.25	4.75	5.25
x_2	1.13	1.50	1.875	2.25	2.62	3.00	3.37
$f(x)$	6.33	9.81	14.07	19.13	24.95	31.56	38.95

b) Fixed m_1 and changing of m_2 resources.

m_2	0	1	2	3	4	6	8
x_1	2.25	2.81	3.37	3.94	4.50	5.63	6.75
x_2	1.13	1.000	0.87	0.75	0.63	0.37	0.13
$f(x)$	6.33	8.91	12.15	16.06	20.64	31.78	45.57

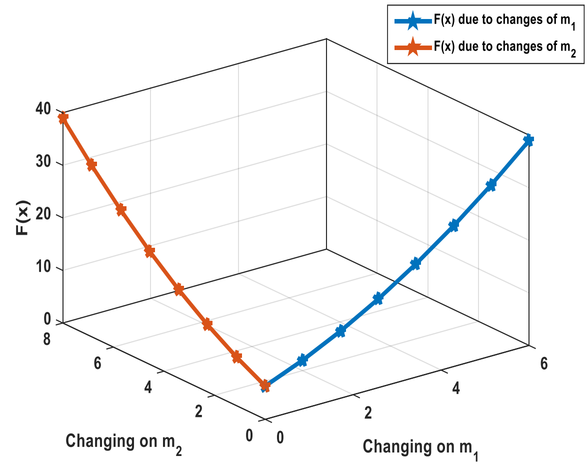


Fig. 5 Overall optimization results of $f(x)$ due to changes of m_1 and m_2 for **Illustrative Example #1.**

V. IMPLEMENTATION USING NONLINEAR PROGRAMMING

A. Mathematical Formulation

The general form of Nonlinear Programming Optimization problem with movable resources m_i can be represented as follows:

$$\begin{aligned} &\text{Minimize} \quad f(x) \\ &\text{subject to} \quad g_1(x) = b_1 + m_1 \\ &\qquad g_2(x) = b_2 - m_1 \\ &\qquad \vdots \\ &\qquad g_{n-1}(x) = b_{n-1} + m_{n-1} \\ &\qquad g_n(x) = b_n - m_n \end{aligned} \tag{12}$$

where $f(x)$ and $g_i(x)$ are nonlinear functions of x.

By applying Lagrange Function to the formulation of (12), we obtain:

$$L(x,\lambda) = f(x) - \lambda_1[g_1(x)-(b_1+m_1)] - \lambda_2[g_2(x) - (b_2 - \cdots - \lambda_{n-1}[g_{n-1}(x)-(b_{n-1}+m_{n-1})] - \lambda_n[g_n(x) - (b_n - m_n)] \tag{13}$$

$$\frac{d\ L(x,\lambda)}{dx_n} = 0 \tag{14}$$

and

$$\frac{dL(x,\lambda)}{d\lambda_n} = 0. \tag{15}$$

Equations (13) to (15) can be solved using Symbolic-based Matlab MuPad Software through matrix manipulation keeping the movable parameters as symbols at all steps of solution.

B. Illustrative Example

We will demonstrate now the concept of introducing the movable resources facilities with the original fixed unchangeable facilities by solving the following nonlinear programming **Illustrative Example #2**.

Original Problem #2:

The original problem without movable resources can be expressed as:

$$\begin{aligned} \text{Minimize} \quad & f(x) = x_1^3 + x_2^2 + x_3^2 \\ \text{subject to} \quad & g_1(x) = x_1^2 + x_2^2 + x_3^2 = 10 \\ & g_2(x) = x_1^3 + x_2^2 + 4x_3^2 = 20. \end{aligned} \quad (16)$$

Modified Problem #2:

The modified problem #2 after adding single movable resource m_1 can be written as:

$$\begin{aligned} \text{Minimize} \quad & f(x) = x_1^3 + x_2^2 + x_3^2 \\ \text{Subject to} \quad & g_1(x) = x_1^2 + x_2^2 + x_3^2 = 10 + m_1 \\ & g_2(x) = x_1^3 + x_2^2 + 4x_3^2 = 20 - m_1. \end{aligned} \quad (17)$$

Following the Lagrange Function formulation described above in (13) to (15) and solving the matrix formulation using Matlab MuPad software, we arrive to the symbolic optimal solution with movable resources as follows:

$$x_1 = \frac{2}{3}$$

$$x_2 = \sqrt{\frac{500}{81} + \frac{5}{3}m_1} \quad (18)$$

and

$$x_3 = \sqrt{\frac{274}{81} - \frac{2}{3}m_1}.$$

Based on the optimal solution results, the changes of optimum solution versus changes in the movable resource m_1 in the range $0 \le m_1 \le 5$, are shown in TABLE 4 and plotted in **Fig. 6**.

TABLE 4 Optimization results of the **Illustrative Example #2** versus the changeable resource m_1

m_1	0	1	2	3	4	5
x_1	0.667	0.667	0.667	0.667	0.667	0.667
x_2	2.484	2.799	3.083	3.342	3.583	3.809
x_3	1.839	1.648	1.431	1.1759	0.846	0.222
$f(x)$	23.382	21.715	20.049	18.383	16.716	15.049

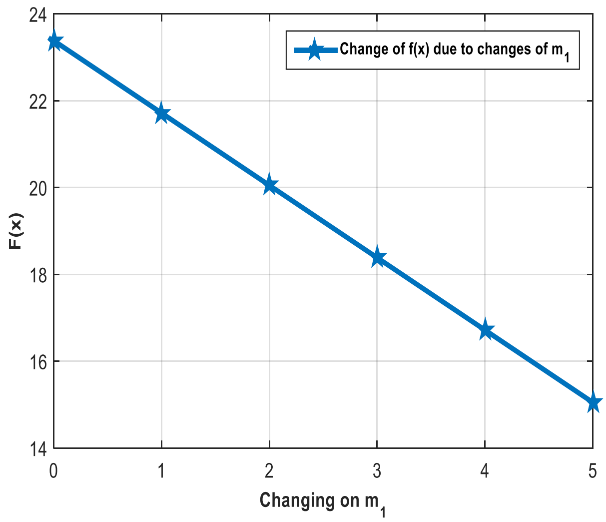


Fig. 6 Optimization results of the **Illustrative Example #2** versus the changeable resource m_1.

VI. IMPLEMENTATION USING MULTI-OBJECTIVE PRGRAMMING

A. Mathematical Formulation

The general form of Multi-objective Programming optimization problem with movable resources m_i can be represented given hereafter.
Consider the following problem with the separate multi-objectives z_i's [12, 13]:

$$\begin{aligned} z_1 &= Minimize\ f_1(x) \\ z_2 &= Minimize\ f_2(x) \\ &\vdots \\ z_n &= Minimize\ f_n(x) \end{aligned} \quad (19)$$

such that $f_i(x)$ are nonlinear or quadratic functions of x.

Using the weighted multi-objective optimization methods, the overall multi-objective function can be expressed as:

$$\text{Minimize Z} = w_1.z_1 + w_2.z_2 + \ldots + w_n.z_n \quad (20)$$

such as the weightings w_i are non-negative and

$$w_{1+} w_n + \ldots + w_n = 1. \quad (21)$$

Accordingly, the general form of Multi-objective Nonlinear Programming optimization problem with movable resources m_i can be expressed now as follows:

$$\text{Minimize} \quad Z = w_1.f_1(x) + w_2.f_2(x) + \ldots + w_n.f_n$$

$$\begin{aligned} \text{subject to} \quad & g_1(x) = b_1 + m_1 \\ & g_2(x) = b_2 - m_1 \\ & \vdots \\ & g_{n-1}(x) = b_{n-1} + m_{n-1} \\ & g_n(x) = b_n - m_n \end{aligned} \quad (22)$$

where $g_i(x)$ are nonlinear or linear functions of x.

The optimization problem of (22) can be similarly solved aso the Nonlinear Programming optimization problem described in (12) following the Lagrange Function approach of (13) to (15).

For multi-objective Quadratic Programming formulations, the solution will follow the typical solution given in (7) as shown before. The same approach could also be extended for Goal Programming Optimization problems [11, 14].

B. Illustrative Example

We will demonstrate now the concept of introducing the movable resources facilities with the original fixed unchangeable facilities by solving the following multi-objective quadratic programming **Illustrative Example #3**.

Original Problem #3:

The original problem without movable resources can be expressed as:

$$\begin{aligned} Minimize\ f(x) = {} & w_1(x_1 - 2x_2 + 4x_3 + x_4 + x_1^2 \\ & + 2x_2^2 + 3x_3^2 + x_4^2 + x_1x_3) \\ & + w_2(x_1^2 + x_2^2 + x_3^2 + x_4^2) \\ \text{subject to} \quad & 3x_1 + 4x_2 - 2x_3 + x_4 = 10 \\ & -3x_1 + 2x_2 + x_3 + 2x_4 = 2 \\ & 2x_1 + 3x_2 - 4x_3 + x_4 = 5 \\ & x_1 + x_2 + x_3 - x_4 = 12. \end{aligned} \quad (23)$$

Modified Problem #3:

The modified multi-objective programming problem of (23) can be re-written after adding the two movable resources m_1 and m_2 as follows:

$$\begin{aligned} Mininize\ \ f(x) = {} & w_1(x_1 - 2x_2 + 4x_3 + x_4 + x_1^2 \\ & + 2x_2^2 + 3x_3^2 + x_4^2 + x_1x_3) \\ & + w_2(x_1^2 + x_2^2 + x_3^2 + x_4^2) \\ \text{subject to} \quad & 3x_1 + 4x_2 - 2x_3 + x_4 = 10 - m_1 \\ & -3x_1 + 2x_2 + x_3 + 2x_4 = 2 + m_1 \\ & 2x_1 + 3x_2 - 4x_3 + x_4 = 5 + m_2 \\ & x_1 + x_2 + x_3 - x_4 = 12 - m_2. \end{aligned} \qquad (24)$$

Equation (24) is a typical Quadratic Programming problem that can be solved by (7), that is: $x^* = Q^{-1}c^T + Q^{-1}A^T(AQ^{-1}A^T)^{-1}(b - AQ^{-1}c^T)$, such as:

$$A = \begin{bmatrix} 3 & 4 & -2 & -1 \\ -3 & 2 & 1 & 2 \\ 2 & 3 & -4 & 1 \\ 1 & 1 & 1 & -1 \end{bmatrix} \qquad (25)$$

$$Q = \begin{pmatrix} w_1 + w_2 & 0 & w_1 & 0 \\ 0 & 2w_1 + w_2 & 0 & 0 \\ 0 & 0 & 3w_1 + w_2 & 0 \\ 0 & 0 & 0 & w_1 + w_2 \end{pmatrix} \qquad (26)$$

$$c = [w_1 \quad -2w_1 \quad 4w_1 \quad w_1\,] \qquad (27)$$

and

$$b = \begin{bmatrix} 10 - m_1 \\ 2 + m_1 \\ 5 + m_2 \\ 12 - m_2 \end{bmatrix}. \qquad (28)$$

Using the Symbolic-based Matlab MuPad Software the above quadratic programming optimization problem can be solved through matrix manipulations.

The optimization results are obtained as functions of the movable resources m_1 and m_2 in symbolic form. Selected forms of solution are illustrated in TABLE 5 and plotted in **Fig. 7 (a, b,c).**

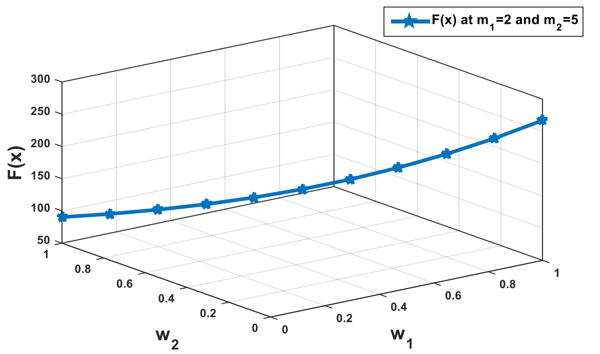
TABLE 5 Selected optimization results of the **Illustrative Example #3** for $m_1 = 2$ and $m_2 = 5$ versus changeable optimization function weighted parameters w_1 and w_2.

w_1	0	0.2	0.4	0.6	0.8	1
w_2	1	0.8	0.6	0.4	0.2	0
x_1	-3.20	-3.4	-3.62	-3.82	-4.02	-4.22
x_2	5.18	5.58	5.98	6.38	6.78	7.18
x_3	-1.9	-2.7	-3.58	-4.38	-5.18	-5.98
x_4	-7.02	-7.2	-7.42	-7.62	-7.82	-8.02
$f(x)$	90.40	107.32	131.55	164.99	209.58	267.23

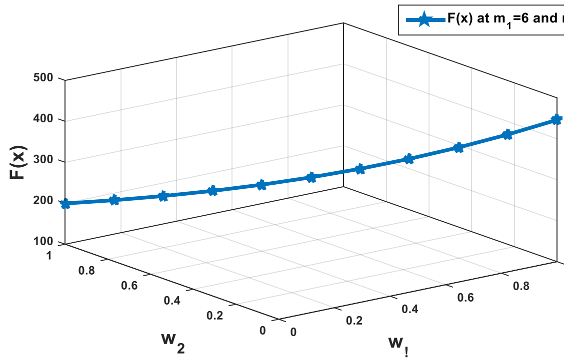
VII. REAL LIFE APPLICATIONS

A. General Applications

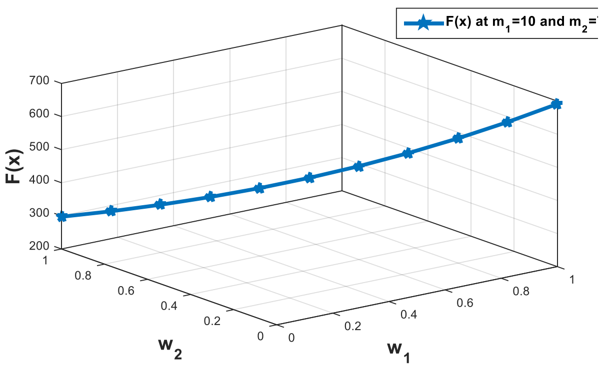
At present, there are many operational networks in real life where some of the resources are changeable in either amount or location. These could be regarded as flexible resources that require different way of handling compared to fixed amount and location resources. Usually the movable and changeable resources are introduced after the construction of the basic fixed network. Nevertheless, it is intended in this research that such notion of movable and changeable resources be considered during the system design process and conceived as a powerful mechanism for adaptively encountering future changing operation environment of the system.



a) $m_1 = 2$ and $m_2 = 5$



b) $m_1 = 6$ *and* $m_2 = 3$.



c) $m_1 = 10$ and $m_2 = 7$.

Fig. 7 Selected optimization results of the Illustrative Example #3 versus changeable weighted parameters w_1 and w_2.

The proposed approach of the flexible reconfigurable networks operation using movable/changeable resources has wide application to real life operational systems of the stand-alone unit or as interconnected multi-units networks. Examples of the applications of these interconnected operational networks with movable/changeable resources are shown in **Fig. 8**, and listed as follows [15]:

1) Irrigation and agriculture drainage networks
2) Electric generation and distribution networks
3) Potable water and wastewater networks
4) Oil and gas pipelines networks
5) Communication and computer networks.
6) Traffic and transportation networks
7) Security and fire protection networks
8) Customer services including financial and health networks.

These applications have many aspects in common. The first aspect is that they are operational and subjected to parameters varying supply and demand. The second aspect is that they could be operated using the suggested Flexible Reconfigurable Networks with movable and changeable resources. The third aspect is that each network cannot operate separately but has to interact with other networks at different levels. All these networks could be provided with additional movable or changeable resources.

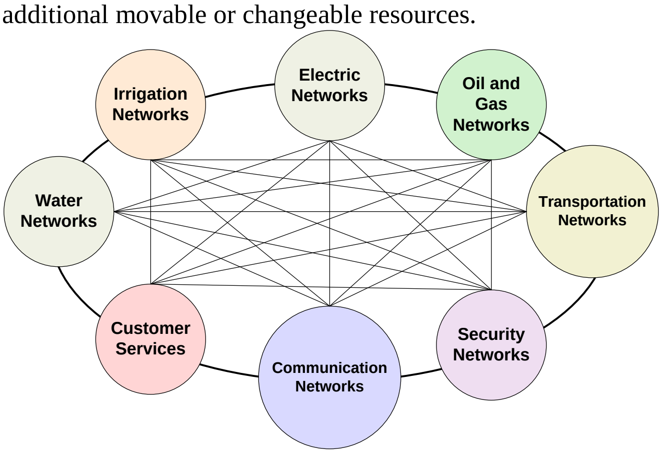


Fig. 8 A diagram showing real life interactions between different operational systems with movable and changeable resources.

Usually, the specifications of movable units are different than the fixed ones as they must be more robust and withstand environmental and mechanical movement problems. Nevertheless, the benefit gained from the flexibility of symbolic-based changing the networks from fixed unchanging ones to more powerful flexible reconfigurable ones could justify any additional costs included in their incorporation in the system configuration [16-18].

C. *A Selected Application Example*

A **t**ypical Flexible Reconfigurable Network in real life application of an irrigation network with 15 pumping stations nodes (12fixed and 3 movable) at El-Nasr Area West of Delta (Egypt) is sketched in **Fig. 9** [19].

The fixed nodes comprise different types of pumping stations of the large, medium and small capacity sizes. The three movable nodes are truck mounted pumping units added to enhance the overall optimal operation of the irrigation network. Such movable stations can be moved within the irrigation pumping network based on irrigation water supply versus demand requirements at each node in the network..

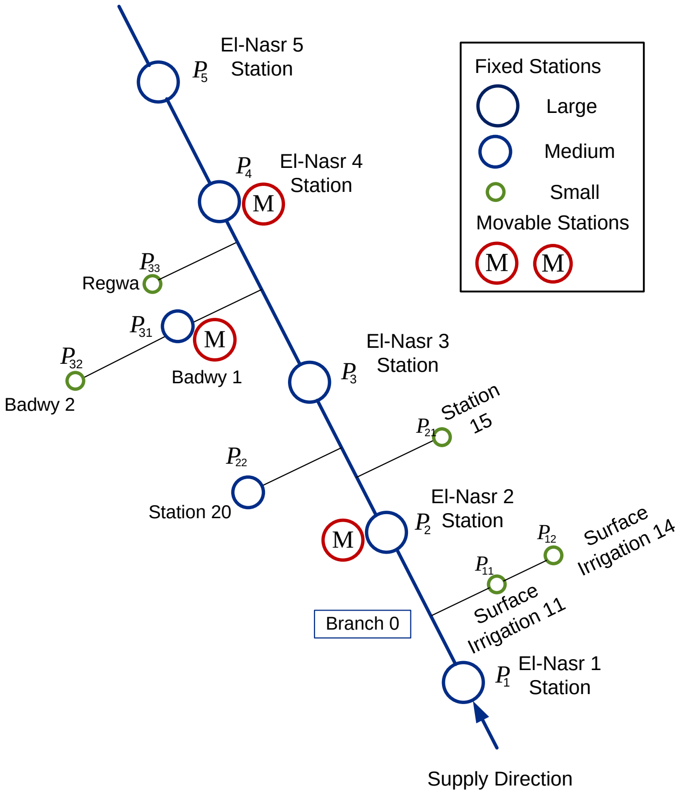


Fig. 9 A **t**ypical real life Flexible Reconfigurable Flexible application of an irrigation network

The optimal operation of this Flexible Reconfigurable Network application could be formulated as a conventional nonlinear multi-objective or goal programming problem. The objectives are based on optimizing overall energy consumption from both fixed and movable pumping nodes. while minimizing economic and environmental losses due to any water demand deficiencies of agriculture area supplied by these nodes. The RHS of system constrains are adaptively generated from time changing supply and demand data resources at various fixed and movable nodes.

VIII. CONCLUSIONS

The notion of ***Flexible Reconfigurable Networks*** introduced in this paper is a new approach that can be used in many optimal applications in real life systems. In general, the term "***reconfigurable***" applied in this work implied to rearrange the elements (after it was configured already) (this is the proposed approach that gives flexible system that can be reconfigured at any time based on the situation).

The implementation and application examples indicate the efficacy of the proposed technique and its strong capability in presenting various decision to make many scenarios for system operation under movable or changeable resource.

For the **proof of concept**, several representable optimization examples are solved in the generic exact symbolic way, through MATLAB MuPAD software addressing symbolic representation and optimization solution. This leads at the end to a general policy could be attained and applied in a flexible manner by manipulating the available flexible movable/changeable components.

The methodology was demonstrated with three applications of Quadratic Programming, Nonlinear Programming, and Multi-objective Programming optimization formulations. The application to such problems are practical to be solved by Matlab MuPad as their solutions steps are carried out through matrix manipulation and simple algebraic calculations which are easy to be implemented through symbolic computations.

The suggested methodology could have many applications in real life systems. Examples of these systems are: the energy generation networks, irrigation water pumping, drinking water networks, the gas networks, wastewater networks, traffic and transportation networks, measuring sensors networks, and service-based networks. Finally, the proposed approach could be extended in a similar symbolic-based fashion to other optimization techniques following the algebraic and matrix methods of solution. This will help in opening the door towards building a new generation of ***Reconfigurable Flexible Networks***.

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